ABSTRACT

This paper presents probabilistic analysis of structural capacity of pre-stressed concrete containments subjected to internal pressure. The conventional design methods for containments are based on allowable stress codes which ensure certain factor of safety between expected load and expected structural strength. Such an approach may give different values of structural reliability in different situations. In recent years, two international round robin exercises have been conducted aimed at predicting the capacity of lined and unlined pre-stressed concrete containments used in nuclear industry. These exercises involved experimental testing and numerical analysis of the models. The first exercise involved ¼ scale steel-lined Pre-stressed Concrete Containment Vessel (PCCV) which was tested at Sandia National Laboratories (SNL) in USA. The second used an unlined containment being tested by the Bhabha Atomic Research Centre (BARC), Tarapur, India. These studies are essentially deterministic studies that have helped validate the analysis methodology and modelling techniques that can be used to predict pre-stressed concrete containment capacity and failure modes. The paper uses these two examples to apply structural reliability method to estimate the probability of failure of the containment.

The two international round robin exercises have already established the ultimate structural collapse mode of the containments under internal pressure loading which indicate that the failure takes place in the general field of the containment wall around mid-height and away from any major structural discontinuities like the penetrations. This is because robust design procedures have been used to avoid structural failure at discontinuities by providing adequate compensation. Based on these experimental studies and the attendant numerical analyses a failure function is presented that assumes first yielding in the hoop direction at mid-height of the cylinder wall. A failure function equating the free-field membrane hoop stress to the hoop strength as a function of cross-sectional area (per unit height) and yield stresses of concrete, rebar, liner plate and tendons is developed.

First Order Reliability Method (FORM) is applied to predict probability of failure of the containments. Probability of failure vs internal pressure is presented for both types of containments. The paper presents a simple method to establish structural reliability of a pre-stressed concrete containment which can be useful for probabilistic safety assessment when considering extreme events that lead to over-pressurisation of the containment.

INTRODUCTION

A pre-stressed concrete containment is an important safety related structure as it acts as one of the final barriers to radioactive release. These structures are normally designed in accordance with the allowable stress codes to sustain the specified loading conditions. However, the compliance with the industry standard allowable stress codes does not give any reliable indication of the probability of failure (Pf) if the
containment is over-pressurised under postulated beyond design basis events. In the past few years, two international round robin exercises have been conducted which have provided valuable test data related to failure under over-pressurisation. The first exercise involved the numerical analysis of the ¼ scale steel-lined Pre-stressed Concrete Containment Vessel (PCCV) with design pressure (Pd) of 0.39MPa which was tested at Sandia National Laboratories (SNL) in USA and has been analysed by Prinja and Shepherd (2003). The second exercise involved the unlined Bhabha Atomic Research Centre (BARC) Containment test model (BARCOM) with Pd of 0.1413 MPa that is being tested by the BARC in Tarapur, India and has been analysed by Kamatam and Prinja (2011). These studies are essentially deterministic studies that have helped validate the analysis methodology and modelling techniques that can be used to predict pre-stressed concrete containment capacity and failure modes. Such deterministic analytical and experimental studies have helped to establish the mode of failure but do not give any indication of Pf. Furthermore, the conventional allowable stress codes used to design such containments also do not provide Pf information. The aim of this paper is to present a simple method to establish structural reliability of a pre-stressed concrete containment which can be useful for probabilistic safety assessment when considering over-pressurisation under extreme events.

FAILSAFE MODE

Both SNL and BARCOM tests have shown that the collapse of the containment structure subjected to internal pressure is not expected to occur soon after the design pressure is exceeded. There is no ‘cliff edge’ but a gradual progressive damage of the containment structure under over-pressurisation which indicates safety margin against collapse. The structure may suffer local failures leading to functional failure well before the ultimate structural collapse. The experiments and the attendant numerical analyses have established the ultimate structural collapse mode of the containments under internal pressure loading which indicates that the failure takes place in the general field of the containment wall around mid-height and away from any major structural discontinuities like the penetrations. This is because robust design procedures have been used that provide adequate compensation and local strengthening to avoid structural failure at discontinuities. Based on these experimental studies and the attendant numerical analyses a failure function is presented that assumes first yielding in the hoop direction at mid-height of the cylinder wall.

In the case of the SNL model shown in Figure 1, the failure location at applied pressure (P) of 3.65 Pd was accurately predicted by the computational model at mid-height of the cylinder in the general area away from the buttress and main penetrations. The BARCOM model is also predicted to fail at mid-height of the cylinder wall as indicated in the deformed shape shown in Figure 2.

![Figure 1. Predicted failure mode of the SNL model (a) FEA results vs (b) test at P=3.65 Pd](image-url)
Figure 2. Predicted response of the BARC model (a) under prestress only and (b) at P=2.89 Pd

Load-deflection curve obtained from the test is compared against that predicted by analysis for the SNL model in Figure 3 at location 14 near the failure location. Note that in the test the internal pressure is released soon after the break but in the analysis the pressure is maintained. The SNL model failed at P/Pd=3.65 in test and was predicted to fail at P/Pd=3.35 in the analysis.

Figure 3. SNL test vs analysis comparison of deflection near failure location

FAILURE FUNCTION

Failure of a containment structure is dictated by the strain levels experienced by the tendons, rebars and the liner following the tensile cracking of the concrete. The first membrane yield is expected to occur in the
hoop direction in the cylinder wall. If the failure state is defined as the tensile cracking of the concrete and yielding of the tendons, rebar and the liner, then the internal pressure at a specific deformed shape is given by:

\[ P = \frac{1}{R}(A_s * F_s + A_c * F_c + A_l * F_l + A_t * F_t) \]  

Where \( A_s, A_c, A_l, A_t \) are cross-sectional areas of the rebar steel, concrete, liner plate and tendons respectively given as area per unit height of the cylinder wall. \( F_s, F_l, F_t \) are yield stress of rebar steel, liner plate and tendons respectively and \( F_c \) is the tensile strength of the concrete. \( R \) is the mid-radius of the cylinder wall.

The failure function 'g' can be written as:

\[ g = PR - (A_s * F_s + A_c * F_c + A_l * F_l + A_t * F_t) \]  

FORM Analysis

If \( Z \) is a function of many basic variables then \( Z = g(x_1, x_2, \ldots, x_n) = 0 \) can be written using Taylor series as:

\[ Z = g(x^*_1, x^*_2, \ldots, x^*_n) + \sum_{i=1}^{n} (x_i - x^*_i) g'_i (x^*_i) + \ldots \]  

where \( g'_i (x^*_i) \) is derivative \( \frac{\partial g}{\partial x_i} \) evaluated at \( x_i = x^*_i \)

\[ Z = k_0 + \sum_{i=1}^{n} k_i x_i \]  

The mean \( \mu_Z \) and standard deviation \( \sigma_Z \) of \( Z \) are given as:

\[ \mu_Z = k_0 + \sum_{i=1}^{n} k_i m_i \]  

\[ \sigma_Z = \left[ \sum_{i=1}^{n} k_i^2 \sigma_i^2 \right]^{1/2} \]  

The reliability index, \( \beta \) is given by

\[ \beta = \frac{\mu_S}{\sigma_Z} \]  

with probability of failure, \( P_f = \phi(-\beta) \) and reliability, \( R = 1 - \phi(-\beta) \)

where \( \phi \) is the standardised cumulative normal distribution.

In structural reliability, eqn (2) can also be written in terms of load (L) and strength (S) terms as follows:

\[ Z = g(x_1, x_2, \ldots, x_n) = S - L \]  

where \( S = (A_s * F_s + A_c * F_c + A_l * F_l + A_t * F_t) \) and \( L = PR \)

If \( \mu_S \) and \( \mu_L \) are mean values and \( V_S \) and \( V_L \) are coefficient of variation (CoV) of the strength and the load terms (S and L) respectively, then the reliability index, \( \beta \) can be written in terms of Central Factor of Safety (\( \eta \)) defined as the ratio of the mean values of S and L terms (\( \eta = \mu_S / \mu_L \)):
The above equation has been used to obtain $P_f$ for various values of the Central Factor of Safety assuming that CoV of both load and strength terms are equal. Figure 4 shows the $P_f$ vs $\eta$ plots for CoV of 0.1 and 0.2. It can be seen that when $V_S = V_L$, the probability of failure is 50% when the load term equals the strength term.

\[
\beta = \frac{(\mu_S - \mu_L)}{\sqrt{\left(\frac{\sigma_S^2}{\mu_S} + \frac{\sigma_L^2}{\mu_L}\right)}} = \frac{(\frac{\mu_S}{\mu_L} - 1)}{\sqrt{\left(\frac{\sigma_S^2}{\mu_S} + \frac{\sigma_L^2}{\mu_L}\right)}} = \frac{(\frac{\mu_S}{\mu_L} - 1)}{\sqrt{\left(\frac{\sigma_S^2}{\mu_S} + \frac{\sigma_L^2}{\mu_L}\right)}}
\]

\[
\beta = \frac{(\eta - 1)}{\sqrt{(V_S^2\eta^2 + V_L^2)}}
\]

Advanced FORM Analysis

The failure function of the containment structure given in eqn (2) has ten variables. When all ten variables are used, the failure function becomes nonlinear and advanced FORM analysis is used following the iterative algorithm recommend by Rackwitz (1976):

1. Guess an initial value of $\beta$ typically starting with $\beta = 3$
2. Set $x_i^* = \mu_i$ for all i. All variables set to their respective mean value $\mu$ at the start.
3. Compute partial derivative of $\frac{\delta \beta}{\delta x_i}$ also known as $a_i$ for all i at $x = x^*$
4. Compute Sensitivity factors, \( \alpha_i = \frac{a_i \sigma_i}{\sum_{j=1}^{n} (a_j \sigma_j)^2} \)

5. Compute new \( x^* \) values using \( x_i^* = \mu_i - (\alpha_i \beta \sigma_i) \)

6. Repeat step 3 to 5 to get stable values of all \( x^* \)

7. Evaluate \( z = g(x_1^*, x_2^*, ..., x_n^*) \)

8. Evaluate \( \frac{dz}{d\beta} \) using \( \frac{dz}{d\beta} = \sum -a_i \cdot \alpha_i \cdot \sigma_i \)

9. \( \beta_{new} = \beta_{old} - Z_n / \frac{dz}{d\beta} \)

10. Compute modified design values \( x_i^* = \mu_i - (\alpha_i \beta \sigma_i) \)

11. Repeat steps 3 to 11 till stable value of \( \beta \) is achieved.

12. Calculate probability of failure, \( P_f = \Phi(-\beta) \)

**PROBABILITY OF FAILURE CALCULATIONS**

The cross-sectional area properties were obtained from the geometric data as follows:-

Steel rebar area/unit height, \( A_s = n_s \cdot \pi \cdot r_i^2 / h_t \)

Where \( n_s \) = number of steel rebars through wall thickness

\( r_i \) is radius of steel rebar and \( h_t \) is the vertical spacing.

Liner area/unit height, \( A_l = \text{thickness of the plate} \times 1 \)

Tendon area/unit height, \( A_t = n_t \cdot \pi \cdot r_t^2 / h_t \)

Where \( n_t \) = number of tendons through wall thickness. \( r_t \) is tendon radius and \( h_t \) is tendon vertical spacing.

Concrete area/unit height, \( A_c = ((r_o - r_i) - (A_l + A_s + A_t)) \)

\( r_o \) and \( r_i \) are outer and inner radii of the wall and mid-radius of the wall, \( R = (r_o + r_i)/2. \)

**SNL Containment Data**

In the SNL model, each tendon is built from 3 wires of 13.7mm diameter so and there were 90 hoop tendons in the wall height of 10750mm giving tendon vertical spacing, \( h_t = 119.4 \text{mm}. \)

With \( n_t = 3 \) and \( r_t = 6.85 \text{mm} \)

\( A_t = (3 \cdot 3.14 \cdot 6.85 \cdot 6.85)/119.4 = 3.70 \text{ mm}^2/\text{mm} \)

There are two hoop rebars (inner and outer) of 22.2mm dia which are vertically spaced at 113mm interval. With \( n_s = 2 \) and \( r_s = 11.1 \text{mm} \)

\( A_s = (2 \cdot 3.142 \cdot 11.1 \cdot 11.1)/113 = 6.85 \text{ mm}^2/\text{mm} \)

\( A_l = \text{thickness of the plate} \times 1 = 1.6 \text{mm}^2/\text{mm} \)

\( A_c = 312.85 \text{ mm}^2/\text{mm} \)

Where \( r_o = 5700 \text{mm}, r_i = 5375 \text{mm} \) and \( R = 5537.5 \text{mm} \)
**BARCOM Containment Data**

The BARCOM model has no liner so $A_l=0$. In the area of failure there are two steel rebars of 12mm dia used as hoop reinforcement through the thickness of the wall at intervals of 200mm. Therefore, $A_s = (2 \times 3.14 \times 6 \times 6) / 200 = 1.1304 \text{ mm}^2 / \text{mm}$

One 24mm dia hoop tendon (horizontal cable) is placed at vertical interval of 110mm giving $A_t = 4.11 \text{ mm}^2 / \text{mm}$. The wall is 188mm thick and goes from elevation level $-2.25 \text{m}$ to $+9.025 \text{m}$ giving wall height of 11275mm.

$r_o = 6376 \text{mm}$, $r_i = 6188 \text{mm}$ and $R = 6282 \text{mm}$.

Therefore, $A_d = 182.76 \text{mm}^2 / \text{mm}$.

The geometric data is summarised in table 1.

The applied internal pressure ($P$) is increased from 0 MPa till probability of failure ($P_f$) of 1.0 is achieved. Mean values and coefficient of variation used for strength and loading variables are given in table 2.

**Results**

Figure 5 presents $P/P_d$ vs $P_f$ curves for both SNL and BARCOM models obtained by using the advanced FORM analysis. Two curves for each model are presented. One in which all ten variables were considered and the other in which the concrete was assumed to be totally damaged due to previous testing and was assigned zero strength.

**DISCUSSION AND CONCLUSIONS**

The available test data and the FEA results (Kamata & Prinja (2011) and Prinja & Shepherd (2003)) showed that the structural response of the pressurised PCCV is indicated by progressive damage in three stages. The first stage up to the design pressure ($P/P_d=1$) is predominantly elastic response and can be predicted with very good accuracy. The second stage involving inelastic response with extensive concrete cracking with local yielding or rupture may lead to loss of functionality (leakage) or breach of pressure boundary. The third stage involves gross deformation leading to the structural collapse. The $P_f$ of this gross structural collapse depends on the amount of steel and concrete used in the design as given in equation 1. The strength terms (given by area x yield stress) for rebar, liner, tendon and concrete are compared in Table 3 for the SNL and BARCOM models along with the design load term (given by $P_d$ x mid-radius of the wall). It can be seen that whilst the overall strength of the two containment models is almost similar, the load term of the BARCOM model is only 40% of the SNL model. The strength/load ratio ($\gamma$) for the SNL is 5.4 but for the BARCOM model it is 9.7 so overall the BARCOM model is nearly twice as strong as the SNL model. This is reflected in the $P_f$ vs $P/P_d$ curves presented in Fig 5. In case of the SNL model, the load term equals the strength term when $P=5.4P_d$ but for the BARCOM model it is when $P=9.7P_d$.

Therefore, the $P_f$ for SNL is 50% when $P=5.4P_d$ but in the test, the SNL model failed catastrophically at $P=3.65P_d$. Catastrophic failure at pressure lower than 5.4Pd could be because of extensive damage to the concrete and the liner due to earlier testing. If similar trend is to be followed then the BARCOM model has to be pressurised beyond 9.7Pd. Such pressurisation in a test may not be easily achieved due to problems with localised failures and leakage.
Table 1 Summary of geometric data for the SNL and BARCOM models

<table>
<thead>
<tr>
<th>Geometric Data</th>
<th>SNL</th>
<th>BARCOM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outside radius of the wall, $r_o$ (mm)</td>
<td>5700</td>
<td>6376</td>
</tr>
<tr>
<td>Inner radius of the wall, $r_i$ (mm)</td>
<td>5375</td>
<td>6188</td>
</tr>
<tr>
<td>Wall thickness (mm)</td>
<td>325</td>
<td>188</td>
</tr>
<tr>
<td>Wall height (mm)</td>
<td>10750</td>
<td>11275</td>
</tr>
<tr>
<td>Mid-radius, $R$ (mm)</td>
<td>5537.5</td>
<td>6282</td>
</tr>
<tr>
<td>No. of tendons through wall, $n_t$</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>Tendon vertical spacing, $h_t$ (mm)</td>
<td>119.4</td>
<td>110</td>
</tr>
<tr>
<td>Tendon radius, $r_t$ (mm)</td>
<td>6.85</td>
<td>12</td>
</tr>
<tr>
<td>Tendon area, $A_t$ (mm²/mm)</td>
<td>3.70</td>
<td>4.11</td>
</tr>
<tr>
<td>No. of rebars through wall, $n_s$</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>Rebar vertical spacing, $h_s$ (mm)</td>
<td>113</td>
<td>200</td>
</tr>
<tr>
<td>Rebar radius, $r_s$ (mm)</td>
<td>11.1</td>
<td>6</td>
</tr>
<tr>
<td>Steel rebar area, $A_s$ (mm²/mm)</td>
<td>6.85</td>
<td>1.13</td>
</tr>
<tr>
<td>Liner plate thickness (mm)</td>
<td>1.6</td>
<td>0</td>
</tr>
<tr>
<td>Liner area, $A_l$ (mm²/mm)</td>
<td>1.6</td>
<td>0</td>
</tr>
<tr>
<td>Concrete area, $A_c$ (mm²/mm)</td>
<td>312.85</td>
<td>182.76</td>
</tr>
</tbody>
</table>

Table 2 Mean values of parameters used for SNL and BARCOM containments

<table>
<thead>
<tr>
<th>Load and Strength Data</th>
<th>Mean Values ($\mu$)</th>
<th>BARCOM/SNL</th>
<th>CoV</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SNL</td>
<td>BARCOM</td>
<td></td>
</tr>
<tr>
<td>Concrete tensile strength, $F_c$</td>
<td>4.4</td>
<td>3.018</td>
<td>69%</td>
</tr>
<tr>
<td>Liner yield, $F_l$</td>
<td>382</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Rebar Yield, $F_s$</td>
<td>465</td>
<td>415</td>
<td>89%</td>
</tr>
<tr>
<td>Tendon yield, $F_t$</td>
<td>1740</td>
<td>1848</td>
<td>106%</td>
</tr>
<tr>
<td>Design Pressure, $P_d$</td>
<td>0.39</td>
<td>0.1413</td>
<td>0.2</td>
</tr>
<tr>
<td>Radius, $R$</td>
<td>5537.5</td>
<td>6282.0</td>
<td>113%</td>
</tr>
<tr>
<td>Concrete Area, $A_c$</td>
<td>312.85</td>
<td>182.76</td>
<td>58%</td>
</tr>
<tr>
<td>Liner area, $A_l$</td>
<td>1.6</td>
<td>0</td>
<td>0%</td>
</tr>
<tr>
<td>Rebar Area, $A_s$</td>
<td>6.85</td>
<td>1.13</td>
<td>17%</td>
</tr>
<tr>
<td>Tendon area, $A_t$</td>
<td>3.70</td>
<td>4.11</td>
<td>111%</td>
</tr>
</tbody>
</table>
Figure 5. Containment P/Pd vs P_f

Table 3 Relative Strength and Load Terms for SNL and BARCOM containments

<table>
<thead>
<tr>
<th>Strength Term</th>
<th>SNL</th>
<th>BARCOM</th>
<th>BARCOM/SNL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rebar (A_r x F_s)</td>
<td>3184.1</td>
<td>469.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Liner (A_c x F_l)</td>
<td>611.2</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Tendon (A_t x F_t)</td>
<td>6439.0</td>
<td>7596.3</td>
<td>1.2</td>
</tr>
<tr>
<td>Concrete (A_c x F_c)</td>
<td>1376.5</td>
<td>551.6</td>
<td>0.4</td>
</tr>
<tr>
<td>Total Strength term, S</td>
<td>11610.8</td>
<td>8617.0</td>
<td>0.7</td>
</tr>
<tr>
<td>Design Load Term (L = Pd x R)</td>
<td>2159.6</td>
<td>887.6</td>
<td>0.4</td>
</tr>
<tr>
<td>Strength/Load Ratio (γ)</td>
<td>5.4</td>
<td>9.7</td>
<td>1.8</td>
</tr>
</tbody>
</table>

SIMPLIFIED METHOD FOR STRUCTURAL RELIABILITY OF CONCRETE CONTAINMENTS

Assuming that the structural collapse of a containment occurs at the mid-height of the wall, the P_f of the containment can be estimated using the simplified procedure presented in Fig 6. All that is required is mean values of the five geometric (A_r, A_c, A_t, R) and four material (F_s, F_l, F_t and F_c) variables to establish the strength/design load ratio (γ). The P_f = 0.5 when the applied pressure, P = γ P_d. P_f at other pressures can be obtained by using either the simple FORM (eqn 10) or advanced FORM for which CoV values for all ten variables are required.
CONCLUSIONS

First Order Reliability Method (FORM) is applied to predict probability of failure of the containments. Probability of failure vs internal pressure is presented for both types of containments (with and without steel liner). Previous studies undertaken as part of the two international roundrobin exercises have established the ultimate structural collapse mode of the containments under internal pressure loading which indicates that the failure takes place in the general field of the containment wall around mid-height and away from any major structural discontinuities. This is because robust design procedures have been used that provide adequate compensation and local strengthening to avoid structural failure at discontinuities. Based on these experimental studies and the attendant numerical analyses a failure function is presented that assumes first yielding in the hoop direction at mid-height of the cylinder wall. It is shown that when the load term (given by P x mid-radius of the wall) equalises the strength terms (given by crosssectional area/unit height x yield stress) for rebar, liner, tendon and concrete then the probability of failure of structural collapse of the containment is 50%. The paper presents a simple method to establish structural reliability of a pre-stressed concrete containment which can be useful for probabilistic safety assessment when considering extreme events that lead to over-pressurisation of the containment.

REFERENCES

